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STRUCTURAL INSTABILITY OF TRANSONIC FLOW OVER AN AIRFOIL

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Inviscid transonic flow over the DSMA523a and Whitcomb airfoils and an airfoil described by a simple algebraic formula has been investigated numerically. The dependence of the flow structure and the lift coefficient on the freestream Mach number M_∞ and the angle of attack α has been studied. The values of M_∞ and α at which steady-state flow becomes unstable have been revealed. The relationship between the flow nonuniqueness occurring in certain ranges of variation of M_∞ and α and the instability has been analyzed.

A local supersonic zone is formed near the airfoil for large subsonic freestream Mach numbers M_∞. If the curvature of the airfoil has a minimum in the vicinity of its midchord, two or more supersonic zones adjacent to its upper and lower sides can be formed. The existence of singular Mach numbers M_s that trigger abrupt changes in the flow structure — either the merging of supersonic regions or their splitting into smaller-size subregions — has been established in [1, 2]. Consideration was given to flow in a channel with a bump modeling an airfoil at the zero angle of attack. Unbounded flow over symmetric airfoils has been investigated in [3] and the relationship between structural instability occurring for $M_\infty = M_s$ and the nonuniqueness of steady-state flow has been analyzed. In this work, an analogous study has been made of two nonsymmetric airfoils developed by NASA and McDonnell Douglas about thirty years ago and of a model nonsymmetric airfoil described by a simple algebraic formula.

Numerical Method. To find the steady-state solutions of the Euler equations we have employed a relaxation method in which unsteady solutions have been calculated using the NSC2KE code based on the finite-volume method [4]. In the code, Van Albada-type limiters are employed [5]. Time integration is carried out using the explicit four-point Runge–Kutta scheme. Numerical modeling of the conditions on the inlet and outlet portions of the external boundary includes the employment of the Steger–Warming flux-splitting scheme [6]. A classical slip condition is employed as the boundary condition on the airfoil. The Mach number $M_\infty < 1$ and the angle of attack α are prescribed at the external boundary of the computational domain. The initial data are the parameters of the freestream homogeneous flow or the steady-state flow obtained for other M_∞ and α.

The structure of transonic flow has been investigated on a nonuniform 733×215 C-grid composed of triangular elements clustering near the airfoil and in the region of shock waves. The external boundary of the computational domain was at a distance $R = 15$ from the airfoil. Test calculations on a coarser grid showed that the error in determining the position of the shock waves significantly increased. On the other hand, the employment of finer grids did not result in a considerably improved accuracy of calculations but substantially increased the amount and time of computations necessary for obtaining steady-state flow. Evaluation of the computational program involved the calculation of transonic flow in the channel and comparison of these results to the data obtained using the ENO2 scheme [2]. Furthermore, we calculated nonsymmetric flow over the airfoil with a flat central part at $\alpha = 0$; the calculations showed that the found dependence of the lift coefficient C_L on M_∞ virtually coincides with the data of [7]. Finally, we calculated flow over the Whitcomb airfoil for the angle of attack $\alpha = 4^o$. When $M_\infty < 0.7$ the dependence $C_L(M_\infty)$ was in good agreement with the results obtained in [8] using the FLUENT code. The noticeable difference in the results for $M_\infty > 0.7$ was due to the increase in the strength of the shock waves and, as a consequence, the separation of the boundary layer, which was disregarded in this work.

Flow about the Whitcomb Airfoil. The first series of calculations was performed for the Whitcomb airfoil [9] at $\alpha = -1.5^o$ in the range $0.760 < M_\infty < 0.785$. They have shown that two supersonic zones are realized on the

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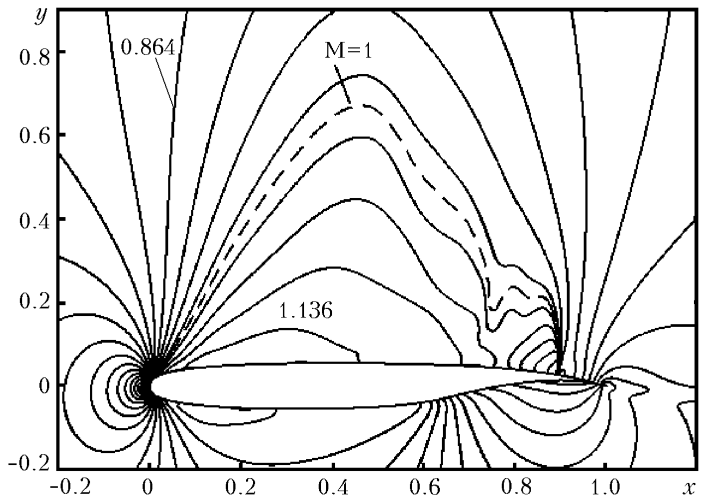

Fig. 1. Mach number isolines over the Whitcomb airfoil for $M_\infty = 0.7748^+$ and $\alpha = -1.5^o$.

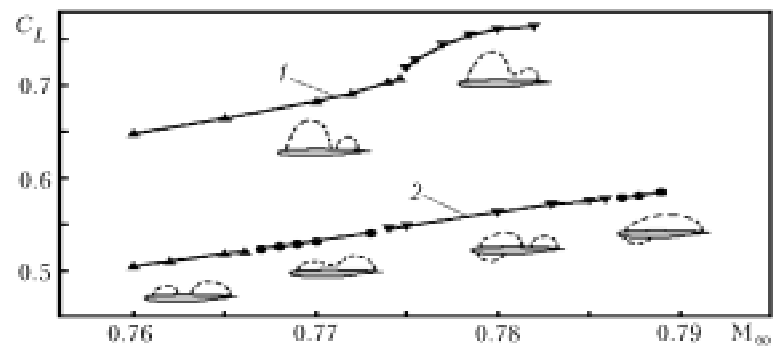

Fig. 2. Lift coefficient C_L vs. M_∞ for the Whitcomb airfoil: 1) $\alpha = -1.5^o$ and 2) -2.5^o.

upper side of the airfoil when $M_\infty = 0.7748$; these zones merge into a single zone when $M_\infty > 0.7748$. Thus, a singular Mach number $M_s \approx 0.7748$ beyond which we have flow restructuring exists. Figure 1 gives the flow pattern that is formed for $M_\infty = 0.7748^+$ (i.e., when M_∞ approaches the singular Mach number from above). On the other hand, if M_∞ tends to M_s from below, a flow pattern with two supersonic regions separated by a distance of 0.03 is realized.

In the vicinity of the singular Mach number M_s, flow is very sensitive to variation of the angle of attack, e.g., for $M_\infty = 0.7749$ a decrease from -1.5^o to -1.6^o in α leads to a splitting of the supersonic region into two subregions. This process evolves with time so that a third concave portion appears in the vicinity of the end of the sonic line; this portion rapidly shifts upstream and merges with the second concave portion, making it deeper, with the result that the supersonic region is split. A return to the initial angle $\alpha = -1.5^o$ leads to a restoration of flow with one supersonic zone.

In the vicinity of the Mach number M_s, flow is also sensitive to small changes in the geometry of the airfoil. In particular, for $M_\infty = 0.7749$ and $\alpha = -1.5^o$ a change in the ordinates of the tail portion $0.9 < x < 1.0$ by $\Delta y(x) = 0.01\,(x - 0.9)$ (so that the height of the airfoil at the endpoint $x = 1$ becomes equal to $y_{\rm end} = 0$ instead of $y_{\rm end} = -0.001$) leads to a restructuring of the flow. The supersonic region on the upper surface is split into two subregions. A return to the initial airfoil ($y_{\rm end} = -0.001$) leads to a merging of these subregions into one region.

At the angle of attack $\alpha = -2.5^o$, we observe three singular Mach numbers $M_s = 0.7665$, 0.7735, and 0.7863 corresponding to the merging and splitting of the supersonic zones on the upper surface (Fig. 2, curve 2). In this figure and in Fig. 4, the regimes of flow with different numbers of supersonic zones near the airfoil are marked by symbols (triangles and dots); these zones are sketched by the dashed curves on the schemes given near the corresponding portions of the plots; the airfoil is shown in gray.

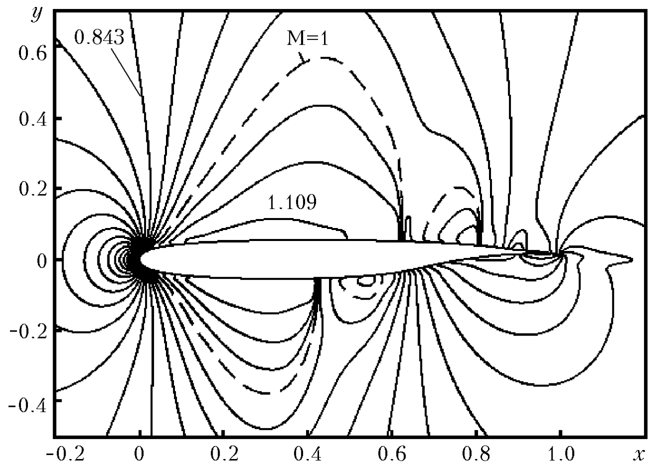

Fig. 3. Mach number isolines at $M_\infty = 0.810$ and $\alpha = -1.1^o$ for the modified Whitcomb airfoil with $y_{end} = 0.012$.

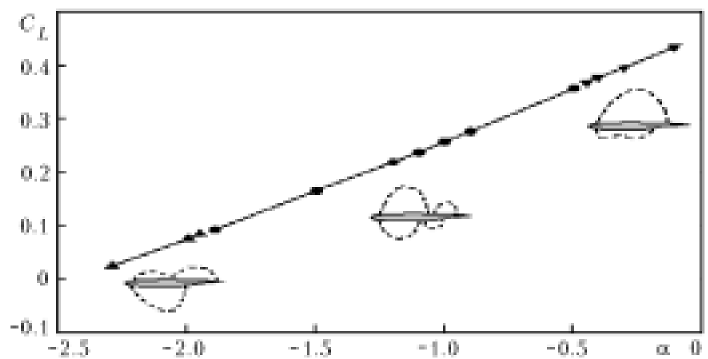

Fig. 4. Lift coefficient C_L vs. angle of attack at $M_\infty = 0.810$ for the modified Whitcomb airfoil with $y_{end} = 0.012$.

Whitcomb Airfoil with a Flap. Let us consider a modification of the Whitcomb airfoil, realized through changing the ordinates of the upper and lower surfaces by $\Delta y(x) = 0.13\ (x - 0.9)$ and $0.9 < x < 1.0$. The height of the endpoint of the airfoil y_{end} increases from 0.001 to 0.012. The above modification in essence corresponds to the rotation of the trailing portion (flap) of length 0.1 through an angle of 7.45^o counterclockwise.

The calculations of flow about such an airfoil with the flap deflected upward have shown that regimes in which two supersonic zones are formed on both the upper and lower surfaces exist (Fig. 3). As M_∞ increases, we observe, at a constant angle of attack, a decrease in the second supersonic zones and their shift downstream followed by their disappearance.

If the Mach number $M_\infty = 0.810$ is fixed, we have the convergence of supersonic zones on the lower surface with increase from -1.1^o to -0.5^o in α and their merging at $\alpha_{s1} \approx -0.4^o$. As α decreases from -1.1^o to -1.4^o, the second supersonic zone shifts downstream, contracts to a point on the airfoil, and disappears. The dependence of C_L on α for this case is presented in Fig. 4. On the upper surface, we have a decrease in the second supersonic zone and its disappearance with increase from -1.1^o to -0.85^o in α and the merging of supersonic zones as α decreases to $\alpha_{s2} \approx -1.9^o$. As is clear from the plot, the dependence of C_L on the angle of attack is linear in character, despite the changes in the flow structure at α_{s1} and α_{s2}.

With a stronger deflection of the flap $\Delta y(x) = 0.15\ (x - 0.9)$, when y_{end} increases to 0.014, the flow velocity on the upper surface decreases and the height of the supersonic zones decreases. On the lower side, the length of the first zone increases, whereas the second supersonic zone contracts to a point and disappears. The downward deflection of the flap so that y_{end} decreases from 0.012 to 0.009, conversely, leads to an expansion of the first supersonic region and disappearance of the second region on the upper surface and to a decrease in the two supersonic regions on the lower surface.

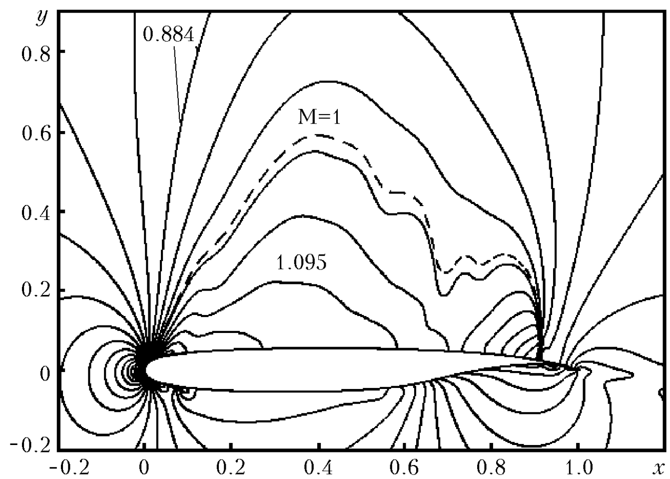

Fig. 5. Mach number isolines over the DSMA523a airfoil for $M_\infty = 0.7742$ and $\alpha = -1.5^o$.

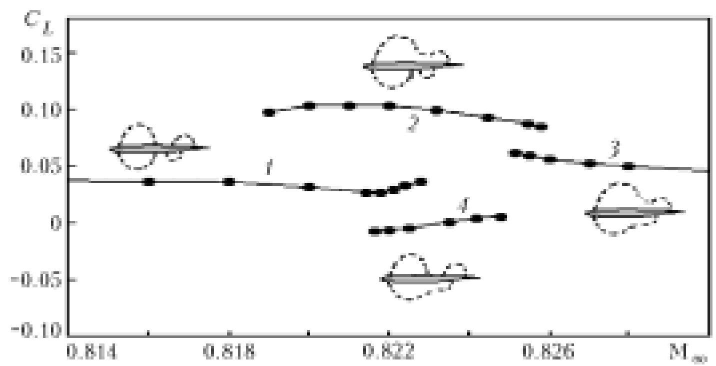

Fig. 6. Lift coefficient C_L vs. M_∞ for the model airfoil at $\alpha = -0.3^o$. Different branches correspond to the flow patterns with different numbers of supersonic zones over the airfoil: 1) two zones on each of the two surfaces; 2) two zones on the lower surface and one zone on the upper surface; 3) one zone on each on the two surfaces; 4) one zone on the lower surface and two zones on the upper surface.

Other Airfoils. The calculations of flow over the DSMA523a airfoil [9] have led to analogous results, e.g., for $M_\infty = 0.7742$ and $\alpha = -1.5^o$ the boundary of the supersonic zone formed on the upper surface has three concave portions (Fig. 5) that are more pronounced here than in Fig. 1. With decrease in M_∞ we have splitting of the supersonic zone into two portions, which is accompanied by an abrupt change in C_L, just as in the case of flow about the Whitcomb airfoil.

The model asymmetric airfoil

$$y(x) = (-1)^m \, 0.06 \sqrt{1 - (2x-1)^4} \, (1 - x^{10})^m , \quad 0 \le x \le 1 ,$$

where $m = 2$ for the upper surface and $m = 3$ for the lower surface, has a smaller curvature in the central part than the Whitcomb and DSMA523a airfoils. Therefore, structural instability is more pronounced here. The calculations of the lift coefficient at $\alpha = -0.3^o$ have shown that the plot of C_L as a function of M_∞ consists of four branches (Fig. 6). Transition from one branch to another can be carried out by prescribing a certain disturbance for the angle of attack at a fixed M_∞ and then by return to $\alpha = -0.3^o$.

On the other hand, if the Mach number M_∞ increases from 0.8140 to 0.8228 at a constant $\alpha = -0.3^o$, we observe the convergence of two supersonic zones on both the upper and lower surfaces. The convergence is very balanced in character, and the state of the flow on both sides of the airfoil turns out to be at the stability boundary for $M_\infty = 0.8228$. If we now increase M_∞ to 0.8233, we observe the merging of supersonic zones on the upper surface. Accordingly the transition from branch 1 of the $C_L(M_\infty)$ plot to branch 2 occurs (Fig. 6). As M_∞ increases further, we have the transition to branch 3 (which is determined by the merging of supersonic zones on the lower surface).

If M_∞ is increased from 0.8224 immediately to 0.8255 (at $\alpha = -0.3^{\circ}$), the merging of supersonic zones on both surfaces occurs and the transition from branch 1 to branch 3 is carried out. As M_∞ gradually decreases from 0.830 to 0.814, the transition from branch 3 to branch 2 (as a result of the splitting of supersonic zones on the lower surface) and next to branch 1 (due to the splitting of supersonic zones on the upper surface) is realized. The transition from branch 1 or 2 to branch 4 can be carried out by decreasing the angle of attack, for example, to $\alpha = -0.6^{\circ}$, followed by restoration of $\alpha = -0.3^{\circ}$.

In the range $0.8190 < M_\infty < 0.8258$, we have nonuniqueness of steady-state flow patterns that are stable to perturbations for all M_∞, except for those values of the Mach number that correspond to the endpoints of branches 2 and 4 and to the right end of branch 1 and the left end of branch 3. An analogous situation has been studied in [3] for certain symmetric airfoils but, unlike Fig. 6, the distance between branches 1 and 3 was very small.

We note that the jumps (obtained above for the Whitcomb and DSMA523a airfoils) in the coefficient C_L, as M_∞ goes across M_s or α goes across α_s, are comparatively small. No M_∞ range in which steady-state flow would be nonunique has been found for these airfoils. At the same time, with allowance for the viscosity of the flow, structural instability can increase in importance (because of the influence of the displacement thickness on the curvature of streamlines) and be comparable to that revealed for the model airfoil.

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NOTATION

C_L, lift coefficient; M_∞, freestream Mach number; M_s, singular Mach number; x, y, Cartesian coordinates; R, chord length; y_{end}, height of the endpoint of the airfoil; Δy, increment of the ordinate; α, angle of attack, deg; α_s, singular value of the angle of attack, deg. Subscripts: end, end; L, lifting force; s, singular; ∞, infinitely remote point.

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